Long Memory in Volatility of T-Bond Futures Markets: A Value-at-Risk Approach

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ABSTRACT

The volatility of T-Bond futures returns are examined by two long memory models, FIGARCH (1, d, 1) and HYGARCH (1, d, 1), with two different innovations distributions: normal and skewed Student-t distributions. The VaRs are calculated by the two fitted models and the empirical evidences are in favor of the HYGARCH (1, d, 1) with skewed student-t innovations based on the Kupiec LR tests. As a result, HYGARCH (1, d, 1) model nicely captures the fat-tail behavior exhibited in the T-Bond futures returns.

INTRODUCTION

There are many styled facts in financial time series documented widely in the existing financial literature, namely unit root property of asset prices, fat tail phenomenon, and volatility clustering. Among them, there is some relationship between fat tail and volatility clustering as persistence effects after shocks to volatility tends to generate returns with high autocorrelation between observations far apart in time (e.g., Ding et al. 1993). Thus, the unconditional and conditional dynamic evolution behaviors of the volatility process in the financial time series seem to be at the heart of the financial research.

The research in estimating and forecasting financial volatility has been widely spread over the last decade. Parametric econometric models for the volatility estimation and forecasting have undergone great performance since the advent of ARCH and GARCH models of Engle (1983) and Bollerslev (1986). A great deal of applications of GARCH type processes has shown the high persistence in the volatility of many financial time series. To take this feature into account, Baillie, Bollerslev, and Millersen (BBM hereafter) (1996) propose the fractionally integrated GARCH (FIGARCH) model, which is an extension of the ARFIMA class of models designed for the conditional mean (Hosking, 1981). The capacity of this type of models to fit stock-return volatility series has been largely analyzed. For example, Ding, Granger and Engle (1993) find no evidence of long memory in daily Standard and Poors 500 returns, but they find some evidence of long memory in the squared returns, which is a proxy of the volatility process of the returns. That documents the long memory features in the volatility process of the stock returns. The long memory models, such as FIGARCH and HYGARCH, become good candidates to analyze the dynamic evolution behavior of the volatility process of the financial time series. With no doubt, it provides a solid foundation to compute the Value-at-Risk (VaR) for the market risk measures.

VaR has become one of the most popular techniques in banking and securities industries to manage their assets market risk in recent years. This is because it provides a simple answer to the following question: with a given probability (say $\alpha$), what is the predicted financial loss of a portfolio over a given time horizon? The answer is the VaR at level $\alpha$, which gives an amount in the currency of the traded assets, and is easily understandable(Giot and Laurent, 2003). Since VaR is defined as the maximum loss can happen given a confidence level, the tail behaviors of the financial time series play an essential role in
calculating VaR for the series. According to the conclusions obtained in the existing literature, which documents that there is long memory property in the volatility of the financial time series, we focus on two popular long memory models.

Duffie and Pan (1997) pointed that we can compare the risks of the different stocks or portfolios via VaR. We can use a simple VaR to evaluate a total risk of portfolio in different investment markets. There are many application of VaR, such as portfolio (Bender (1995) computed the VaR of the portfolio of stocks and bonds via historical method and Monte carlo method, Duarte (1997)), exchange rates and financial assets (Alexander and Leigh (1997), Duffie and Pan (1997)), option ( Estrella et al. (1994), Pritsker (1997), EJ-Jahel, Perraudin and Sellin (1999), Fong and Lin (1999)) and forward contracts (Linsmeier and Pearson (1996)).

In this paper, we investigate the in-sample goodness-of-fit and out-of-sample performances of the two long memory models with different innovation distributions, namely, normal and skewed Student-t distributions. The empirical results show that the in-sample and out-of-sample forecasting performances of the HYGARCH (1, d, 1) with skewed Student-t innovations distribution are better among the models we have executed. Accordingly, in calculating the in-sample VaRs, the HYGARCH (1, d, 1) with skewed Student-t innovations distribution model captures the fat-tail behavior of the T-Bond futures returns based on the Kupiec LR failure rate test since most of the p-values are larger than those in FIGARCH (1, d, 1). Moreover, the results obtained here is contrary to those in Inui, Mijima, and Kitano (2003), who point out that as the confidence level increases, the significant positive bias for VaR values calculations with a fat-tail distribution increases. We do not observe the phenomenon that in-sample VaR calculations are subject to a significant positive bias in the T-Bond futures series.

Nevertheless, for out-of-sample VaR computing, the empirical results show some differences. While the skewed Student-t HYGARCH (1, d, 1) models perform much better for most of the $\alpha$ quartiles both for the long the short position. The empirical results for the normal HYGARCH (1, d, 1) show some interesting results. Unlike the FIGARCH (1, d, 1) model with normal innovations distribution, the normal HYGARCH (1, d, 1) model performs quite well since none of the failure rates are rejected for all the ranges of $\alpha$ quantiles. Whereas the p-values calculated for the empirical failure rates are all smaller than those calculated by HYGARCH (1, d, 1) with skewed Student-t innovations distributions.

We conclude that no matter for the in-sample or out-of-sample VaR calculations for the long and short position, HYGARCH (1, d, 1) model with skewed Student-t innovation distribution is adequate to capture the fat-tails exhibited in the series.

THE MODEL

A. FIGARCH (1, d, 1) model

Ballie, Bollerslev, and Mikkelsen (1996) introduce the Fractional Integrated GARCH (FIGARCH) process to recover the long memory observed in the volatility of stock and exchange rate return series, and the model also fills the gap between short and complete persistence. In contrast to an I (0) time series in which shocks die out at an exponential rate, or an I (1) series in which there is no mean reversion, shocks to an I (d) time series with 0 < d < 1 decay at a slow hyperbolic rate.

The FIGARCH (1, d, 1) model can be defined as

$$[\phi(L)(1-L)^d]\varepsilon_t^2 = \omega + [1-\beta(L)](\varepsilon_t^2 - \sigma_t^2)$$  \hspace{1cm} (1)
The fractional differencing operator \((1 - L)^d\) is defined as:

\[
(1 - L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(d + 1) L^k}{\Gamma(k + 1) \Gamma(d - k + 1)} = 1 - dL - \frac{1}{2} d(1 - d)L^2 - \frac{1}{6} d(1 - d)(2 - d)L^3 - \ldots
\]

(2)

where \(c_1(d) = d\), \(c_2(d) = \frac{1}{2} d(1 - d)\), etc.

B. HYGARCH (1, d, 1) Model

There are some bias problems with FIGARCH models. The unexpected behavior of the FIGARCH model may be due less to any inherent paradoxes than to the fact that the unit-amplitude restriction, appropriate to a model of levels, has been transplanted into a model of volatility. In a more general framework, there are good reasons to embed it in a class of models in which such restrictions can be tested, and also to adhere to the approach of modeling amplitude and memory as separate phenomena.

In view of these considerations, Davidson (2004) develops a new model, the “hyperbolic GARCH,” or HYGARCH (1, d, 1) model. Consider the form

\[
\theta(L) = 1 - \frac{\delta(L)}{\beta(L)} (1 + \alpha((1 - L)^d - 1))
\]

(3)

where \(\alpha \geq 0\), \(d \geq 0\). Note that, provided that \(d > 0\),

\[
S = 1 - \frac{\delta(1)}{\beta(1)} (1 - \alpha)
\]

(4)

The FIGARCH and stable GARCH cases correspond to \(\alpha = 1\) and \(\alpha = 0\), respectively. The hypothesis of either of these two cases might be tested. However, in the latter case the parameter \(d\) is unidentified, which poses a well known problem for constructing hypothesis tests. Therefore, when \(d = 1\), the equation (3) will reduce to

\[
\theta(L) = 1 - \frac{\delta(L)}{\beta(L)} (1 - \alpha L) \quad \alpha \geq 0
\]

(5)

In other words, when \(d = 1\), the parameter \(\alpha\) reduces to an autoregressive root. Hence the model becomes either a stable GARCH or IGARCH, depending on whether \(\alpha < 1\) or \(\alpha = 1\).

When \(d\) is not too large, this model will correspond closely to the case

\[
\theta(L) = 1 - \frac{\delta(L)}{\beta(L)} (1 - \alpha \phi(L))
\]

(6)

where

\[
\phi(L) = \sigma(1 + d)^{-1} \sum_{j=1}^{\infty} j^{-1-d} L^j \quad d > 0
\]

(7)

and \(\sigma(\cdot)\) is the Riemann zeta function. The Riemann zeta function \(\sigma(\cdot)\) is defined for any complex number \(s\) with real part which is greater than one by the Dirichlet series:\n
\[
\sigma(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}.
\]
**A. VaR model**

In this study, the one-step ahead VaR is computed with the result of estimated models and its assigned distribution. The one-step-ahead forecast of the conditional mean $\hat{\mu}_t$ and conditional variance $\hat{\sigma}_t^2$ is computed conditional on past information. The VaRs of $\alpha \%$ quantile for long and short trading positions are computed as follows. Here we only present model with the skewed Student-t innovations distribution.

Let

$$R_t = \mu_t + \epsilon_t,$$

and

$$\mu_t = \mu + \sum_{i=1}^{m} \psi_i R_{t-i} + \sum_{j=1}^{n} \theta_j \epsilon_{t-j}$$

where $\epsilon_t = z_t \sigma_t$ follows the FIGARCH (1, d, 1) or HYGARCH (1, d, 1) process, $z_t \sim i.i.d. N(0,1)$, $R_t$ series follows the ARMA $(m, n)$ process. Under the skewed Student-t distribution, for long and short trading positions are:

$$\hat{VaR}_{t,L} = \hat{\mu}_t + st_{\alpha}(\nu, k) \hat{\sigma}_t,$$

and

$$\hat{VaR}_{t,S} = \hat{\mu}_t + st_{1-\alpha}(\nu, k) \hat{\sigma}_t$$

where $st_{\alpha}(\nu, k)$ is the left quantile at $\alpha \%$ for the skewed Student-t distribution.

**B. Kupiec LR tests**

The VaRs of different pre-specified level $\alpha$ ranging from 5% to 1% are computed and their performance is evaluated by computing their failure rate for the futures return series. The failure rate criterion is widely applied in studying the effectiveness of VaR models. The definition of failure rate is the proportion of the number of times the observations exceed the forecasted VaR to the number of all observations. The standard we use to judge the performance of VaR model is to assess the difference between the pre-specified VaR level and the failure rate. If the failure rate is very close to the pre-specified VaR level, we could conclude that the VaR model is specified very well.

Most empirical literatures in the field of VaR only evaluate the performance of VaR models by comparing the absolute value of failure rate. For the purpose of testing VaR models more precisely, we adopt the Kupiec LR test (Kupiec, 1995) to test the effectiveness of our VaR models. Denote the failure rate as $f$ which is equal to the ratio of the number of observations exceeding VaR ($x$) to the number of total observations ($T$) and pre-specified VaR level as $\alpha$. The statistic of Kupiec LR test is given by:

$$LR = 2 \left\{ \log \left[ f^x \left( 1-f \right)^{T-x} \right] - \log \left[ \alpha^x \left( 1-\alpha \right)^{T-x} \right] \right\}$$

which is distributed as chi-square distribution with 1 level of freedom and is used to test the null hypothesis that the failure rate equals the pre-specified VaR level $\alpha$. 

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DATA DESCRIPTION AND EMPIRICAL RESULTS

A. Contents of the data and price limits

The futures prices of the nearest T Bond contract were obtained from Data Stream database for every trading day transaction carried out from Oct. 4, 1982 to Dec. 31, 2004. The T-Bond futures prices continued rally and turned bullish trend since 1982. Since there is a price limit in the T-Bond futures market, we have deleted 41 observations which hit the price limits, including Oct 20, 1987. On that date, the futures price hit the price limit of three points. The price limit was removed since Aug. 27, 2000.

We only consider the nearest month futures contract, but the criteria must be set in order to create a continuous futures prices time series. We use an automatic roll of the futures contracts to determine the appropriate roll date for contracts based upon total daily tick count. A roll of contracts will occur on the close of the day in which the next contract’s daily tick count exceeds the current contracts. The price index series, \( R_t \), is differenced in the logs to create the raw prices change series.

B. Estimation Results for FIGARCH (1, d, 1) and HYGARCH (1, d, 1)

The empirical results for the long memory models are estimated with the help of the G@RCH 3.0 package provided by Doornik and Ooms (2001).

We employ the FIGARCH (1, d, 1) and HYGARCH (1, d, 1) models to estimate the long memory of the series in this paper. From the results in Tables 1 and 2, the empirical evidences show that the estimated coefficients for the parameters are statistically significant and satisfy the nonnegative conditions for all the coefficients. The in-sample diagnostic tests all evidence that the fitted model is adequate. The fractional integration parameter estimated by the FIGARCH (1, d, 1) model, \( \hat{d} \)-Figarch, is significant different from zero. During the more tranquil sample period from 1982 to 2004, the \( \hat{d} \)-Figarch estimated values range from 0.3339 and 0.4034 for normal and skewed Student-t innovations, respectively. Whereas the \( \hat{d} \)-Figarch estimated values by HYGARCH (1, d, 1) range from 0.1223 and 0.1428, respectively. The empirical evidences lend strong support that there is long memory properties in the volatility process of the T Bond futures returns. The empirical evidence gives the result that the data exhibits “leverage effect” since the asymmetric parameters are statistically significant different from zero. That suggests that the innovations follow skewed Student-t distribution.

<table>
<thead>
<tr>
<th>Table 1: FIGARCH (1, d, 1) parameters estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distributions</td>
</tr>
<tr>
<td>d-Figarch</td>
</tr>
<tr>
<td>(\alpha)</td>
</tr>
<tr>
<td>(\beta)</td>
</tr>
<tr>
<td>Student(DF)</td>
</tr>
<tr>
<td>(\hat{\alpha}) = Asymmetry</td>
</tr>
<tr>
<td>(\hat{\nu}) = Tail</td>
</tr>
<tr>
<td>Log-likelihood</td>
</tr>
<tr>
<td>AIC</td>
</tr>
<tr>
<td>Ljung-Box (Q^2(10)) [p-value]</td>
</tr>
<tr>
<td>ARCH(10) [p-value]</td>
</tr>
</tbody>
</table>

Note: Standard errors are reported in parentheses. The log-likelihood is the maximized value of the log likelihood function. AIC is the Akaike Information Criterion. \( Q^2(10) \) is the Ljung-Box Q-statistic of the order 10 on the squared standardized residuals. ARCH(10) is the ARCH LM test up to order 10 for residuals. * is 1% significant level.
Table 2: HYGARCH (1, d, 1) parameters estimated

<table>
<thead>
<tr>
<th>Distributions</th>
<th>Normal</th>
<th>Skewed Student-t</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-Figarch</td>
<td>0.1223**(0.0612)</td>
<td>0.1428**(0.0718)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.3139*(0.0422)</td>
<td>0.3069*(0.0499)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.5415*(0.0579)</td>
<td>0.5734*(0.788)</td>
</tr>
<tr>
<td>Student(DF)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\hat{\alpha}) = Asymmetry</td>
<td></td>
<td>-0.0543*(0.0179)</td>
</tr>
<tr>
<td>(\hat{\nu}) = Tail</td>
<td></td>
<td>6.2040*(0.5330)</td>
</tr>
<tr>
<td>Log (\alpha)</td>
<td>0.5692(0.5869)</td>
<td>0.5272(0.5549)</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>-31227.602</td>
<td>-31111.220</td>
</tr>
<tr>
<td>AIC</td>
<td>11.1257</td>
<td>11.0850</td>
</tr>
<tr>
<td>Ljung-Box (Q^2(10)) [p-value]</td>
<td>10.9377 [0.2052]</td>
<td>14.6281 [0.0667]</td>
</tr>
<tr>
<td>ARCH(10) [p-value]</td>
<td>1.1123 [0.3483]</td>
<td>1.4420 [0.1550]</td>
</tr>
</tbody>
</table>

Note: Standard errors are reported in parentheses. The log-likelihood is the maximized value of the log likelihood function. AIC is the Akaike Information Criterion. \(Q^2(10)\) is the Ljung-Box Q-statistic of the order 10 on the squared standardized residuals. ARCH(10) is the ARCH LM test up to order 10 for residuals. * is 1% significant level. ** is 5% significant level.

C. VaR Computations

We employ the FIGARCH (1, d, 1) and HYGARCH (1, d, 1) models to compute the VaR values. Not only in-sample VaR values are computed to examine the estimated model’s goodness-of-fit ability but also out-of-sample VaR values are computed to evaluate the forecasting quality of the estimated model. The model is tested with a VaR level \(\alpha\) from 5% to 1%, and its performance is evaluated by computing the failure rate for the futures returns. If the VaR models were specified perfectly, the failure rate would equal to the pre-specified VaR level \(\alpha\). In other words, the more the failure rate approaches the pre-specified VaR level \(\alpha\), the more the VaR model helps investors to forecast their possible trading losses correctly. In this article, we use Kupiec’s LR test to examine the performance of the VaR model. The model with skewed Student-t innovation distributions is adopted to compute the in-sample VaR values both for the long and short positions.

In-sample VaR computations

The results for the in-sample VaR computations of FIGARCH (1, d, 1) and HYGARCH (1, d, 1) models are collected in Tables 3 and 4. It contains the failure rates computed and their corresponding Kupiec’s LR tests. If the VaR model is estimated accurately, it should explain the actual observations very well. Accordingly, the failure rate should be equal to the pre-specified VaR level \(\alpha\), and the Kupiec’s LR test would not reject its null hypothesis (failure rate equals to \(\alpha\)).

Among different innovations distribution, most of the empirical failure rates are rejected by Kupiec LR tests not only in FIGARCH but also in HYGARCH models at 5% significant level. Most of the p-values for the skewed Student-t innovations distributions are larger than those calculated from the models under normal distribution. The interesting result is that we do not observe the fact documented in the existing literature that in-sample VaRs are subject to a significant positive bias. That is, p-values of the empirical failure rate calculated by models with skewed Student-t innovations distributions are larger than those calculated by the model with normal innovations at different \(\alpha\) quantiles. From the results of in-sample VaR computation, we can arrive at the following conclusions: the skewed Student-t HYGARCH model predict critical loss more accurate than the models with normal distribution, respectively, not only for the long position but also for the short position.
### Table 3: FIGARCH In-Sample VaR Results and LR test

<table>
<thead>
<tr>
<th>Short position</th>
<th>Long position</th>
</tr>
</thead>
<tbody>
<tr>
<td>α Quantile</td>
<td>Failure rate</td>
</tr>
<tr>
<td>0.95</td>
<td>0.9523</td>
</tr>
<tr>
<td>0.975</td>
<td>0.9736</td>
</tr>
<tr>
<td>0.99</td>
<td>0.9884</td>
</tr>
</tbody>
</table>

Note: Failure rates and Kupiec LR test statistics for in-sample VaR results. ** is 5% significant level

### Table 4: HYGARCH In-Sample VaR Results and LR test

<table>
<thead>
<tr>
<th>Short position</th>
<th>Long position</th>
</tr>
</thead>
<tbody>
<tr>
<td>α Quantile</td>
<td>Failure rate</td>
</tr>
<tr>
<td>0.95</td>
<td>0.9449</td>
</tr>
<tr>
<td>0.975</td>
<td>0.9746</td>
</tr>
<tr>
<td>0.99</td>
<td>0.9914</td>
</tr>
</tbody>
</table>

Note: Failure rates and Kupiec LR test statistics for in-sample VaR results. ** is 5% significant level

### Out-of-sample VaR computations

In the above in-sample VaR computation, the models estimated from estimation sample are used to compute in-sample VaR, and by comparing these VaR with estimation sample we just know the “past” performance of these VaR models. The real contribution of VaR computation is its forecasting ability, which provides investors or financial institutes with the information about what the biggest loss they will incur is. In this subsection, we show the empirical results in forecasting ability of the VaR models. The out-of-sample VaR is one-step-ahead forecast, which means that the VaR of the \((t+1)\) day is computed conditional on the available information on the \(t\) day. We compute 252 out-of-sample VaRs for T Bond returns. Just similar to in-sample VaR analysis, these out-of-sample VaRs are compared with observed close prices and both results are recorded for latter evaluation using Kupiec’s LR test. The empirical results are shown in Tables 5 and 6.

Most of the models are doing well in calculating the out-of-sample VaR by the Kupiec LR tests. The most interesting thing is that even normal HYGARCH \((1, d, 1)\) also performs well since none of the empirical failure rates are rejected. FIGARCH and HYGARCH models with Student-t and skewed Student-t innovations distributions perform well. After careful investigation for the empirical results from Tables 5 and 6, no matter the long position and short position, the skewed Student-t HYGARCH model performs better.
CONCLUDING REMARKS

The volatility process of the T Bond futures returns is investigated during July 1, 1982 to Dec. 31, 2004 by employing the FIGARCH (1, d, 1) and HYGARCH (1, d, 1) models with normal, Student-t, and skewed Student-t innovations distributions. Furthermore, based on the Kupiec’s LR test, the empirical evidences show that the failure rate implied by the HYGARCH VaR model is more close to the pre-specified confidence level. Thus, the HYGARCH (1, d, 1) with skewed Student-t innovation distribution performs better to calculate the VaR measures.

<table>
<thead>
<tr>
<th>VaR results under normal distribution - FIGARCH</th>
<th>VaR results under skewed student-t distribution – FIGARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short position</td>
<td>Long position</td>
</tr>
<tr>
<td>α Quantile Failure rate Kupiec P-value</td>
<td>α Quantile Failure rate Kupiec P-value</td>
</tr>
<tr>
<td>0.95  0.9623  1.7485  0.1861</td>
<td>0.05  0.0635  1.7861  0.1814</td>
</tr>
<tr>
<td>0.975 0.9762  0.0298  0.8630</td>
<td>0.025 0.0476*  8.3957  0.0038**</td>
</tr>
<tr>
<td>0.99  0.9901  0.0003  0.9857</td>
<td>0.01  0.0337*  17.706  0.2577e-005**</td>
</tr>
</tbody>
</table>

Note: Failure rates and Kupiec LR test statistics for out-of-sample VaR results. ** is 5% significant level

Table 6: HYGARCH Out-of-Sample VaR Results and LR test

<table>
<thead>
<tr>
<th>VaR results under normal distribution - HYGARCH</th>
<th>VaR results under skewed student-t distribution – HYGARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short position</td>
<td>Long position</td>
</tr>
<tr>
<td>α Quantile Failure rate Kupiec P-value</td>
<td>α Quantile Failure rate Kupiec P-value</td>
</tr>
<tr>
<td>0.95  0.9642  1.1974  0.2738</td>
<td>0.05  0.0595  0.4547  0.5001</td>
</tr>
<tr>
<td>0.975 0.9761  0.0148  0.9029</td>
<td>0.025 0.0317  0.4341  0.5100</td>
</tr>
<tr>
<td>0.99  0.9881  0.0870  0.7679</td>
<td>0.01  0.0119  0.0870  0.7679</td>
</tr>
</tbody>
</table>

Note: Failure rates and Kupiec LR test statistics for out-of-sample VaR results.

REFERENCES


Inui, K., M. Kijima, and A. Kitano, (2003), VaR is subject to a significant positive bias, working paper.

